

Drag force in SYM plasma with B field from AdS/CFT

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ABSTRACT: We investigate drag force in a thermal plasma of $N=4$ super Yang-Mills theory via both fundamental and Dirichlet strings under the influence of non-zero NSNS B -field background. In the description of AdS/CFT correspondence the endpoint of these strings corresponds to an external monopole or quark moving with a constant electromagnetic field. We demonstrate how the configuration of string tail as well as the drag force obtains corrections in this background.

KEYWORDS: AdS-CFT Correspondence, Gauge-gravity correspondence, Non-Commutative Geometry.

Contents

1. Introduction	1
2. Heavy monopole probe in QGP	2
2.1 Dragging monopole in the electric field	3
2.2 Dragging monopole in the magnetic field	6
3. Heavy quark probe in QGP	6
3.1 Dragging quark in non-commutative space	7
4. Summary and discussion	9

1. Introduction

At the experiments of Relativistic Heavy Ion Collision (RHIC), collisions of gold nuclei at 200 GeV per nucleon are about to produce a strongly-coupled quark-gluon plasma (QGP), which behaves like a nearly ideal fluid [1]. While the perturbative calculation can not be fully trusted in this strongly-coupled regime, there are increasing amount of interests in calculation of hydrodynamical transport quantities via the use of AdS/CFT correspondence [2, 3]. It is hoped that this line of research will eventually make contact with experiment result from RHIC. As it was first proposed [4], the large N limit of $\mathcal{N} = 4$ super Yang-Mills field theories in four dimensions include in their Hilbert space a sector describing supergravity on the $AdS_5 \times S^5$ background. The curvature of the sphere and the AdS space in units of $\sqrt{\alpha'}$ is proportional to $1/\sqrt{N}$, therefore the solutions can be trusted as long as N is large. In this paper, we are interesting in probing this strongly-coupled QGP by some *theoretically* possible particles, in particular a heavy quark or a heavy monopole. This corresponds respectively to dragging an attached fundamental or Dirichlet string tail cross the whole AdS bulk. Finite temperature configurations in the decoupled field theory correspond to black hole configurations in AdS spacetimes. In particular, there is a linear relation between the size of horizon and Hawking temperature for large AdS black holes. It is suggested by the holographic principle that properties of this strong interaction for probe particles with a hot plasma of gluons and quarks are in part or fully reflected on the dual gravitational background. Apart from the vacuum AdS, or correspondingly super conformal field theory on the boundary, introduction of black hole is hoped to better describe a realistic QCD model on the dual field theory side, though the correspondence is so far not fully satisfied yet.

A probe particle can be prepared by separating a pair of particle-antiparticle in the deconfinement phase to far distance, then it is enough to consider just one single particle

with a string tail all the way up to the black hole horizon. Description of the other particle in the same pair is just a mirror to this. Energy put to this single particle, if any, must either alter the kinetic energy, or dissipated into the surrounding plasma via strong interaction. In particular the latter can be holographically described as dissipated energy being dumped into the black hole via the string energy-momentum current.

Energy loss of a heavy quark moving through $\mathcal{N} = 4$ super Yang-Mills thermal plasma has been extensively studied recently [5]. In particular, the drag force is derived in the context of AdS/CFT to model the effective viscous interaction [6, 7], later it is generalized to a rotational black hole or couple to dilaton field [8–10]. Recently drag force of a comoving quark-antiquark pair is also considered in [11].

In this note, we investigate the drag force via both fundamental and Dirichlet strings under the influence of non-zero NSNS B -field background. We expect that it also sheds some light to real QGP under an external electromagnetic field. From the viewpoint of string theory, B -field is the gauge field to which a string can couple. However it is not clear how does it affects the movement of string endpoint on the AdS boundary, because it couples to F1 and D1 strings in a different way. This paper is organized as follows. In section 2 we calculate the drag force of an external monopole moving in a thermal plasma with constant electromagnetic field in the $\mathcal{N} = 4$ super Yang-Mills theory. In section 3 we consider an external quark in a *noncommutative* super Yang-Mills theory at finite temperature. In section 4, we give a summary and discussion.

2. Heavy monopole probe in QGP

In this section, we consider a monopole which moves in constant speed, and introduce a constant B -field of either electric-type E or magnetic-type H along the x^1 and x^2 direction. Because only the field strength is involved in equations of motion, our ansatz is still a good solution to supergravity. For the dual field theory, this is the minimal setup to investigate on the B -field correction. A simple relevant geometry is a AdS_5 Schwarzschild black hole with constant B -field,

$$\begin{aligned}
 ds^2 &= \mathcal{H}^{-1/2}(-hdt^2 + dx_1^2 + dx_2^2 + dx_3^2) + \mathcal{H}^{1/2}h^{-1}\frac{dr^2}{r^2}, \\
 B &= Edt \wedge dx_1 + Hdx_1 \wedge dx_2, \\
 h &= 1 - \frac{r_H^4}{r^4}, \quad \mathcal{H} = \frac{L^4}{r^4},
 \end{aligned}
 \tag{2.1}$$

where the AdS radius as well as S^5 is given by $L^2 = \sqrt{\lambda\alpha'}$, and the horizon is at $r = r_H$, which is related to temperature by $T = r_H/\pi L^2$ for large black hole. We have introduced constant NS-NS antisymmetric field $B_{01} = E$ and $B_{12} = H$. This is still a solution to the type IIB supergravity. Since only the NS-NS field strength appears in all the equations of motion, it is trivially qualified for $dB_{(2)} = 0$. Here the B field is identified with the field strength of a $U(1)$ field on the boundary. We take the B field is so small that the metric is not deformed by the B field and is the one describing an AdS_5 Schwarzschild black hole. Thus the dual field theory is the ordinary super Yang-Mills theory with finite temperature.

We might regard it as a weak U(1) field limit in which back reactions to the bulk geometry is ignored. Even in such a simplification one might catch some features about drag force of monopole under the influence of constant field strength and this is indeed the case as we will see below.

In the AdS/CFT prescription, monopole corresponds to the endpoint of an open D-string in the bulk supergravity. We may describe such a string attached to the monopole by

$$x_1 = v_1 t + \xi_1(r), \quad x_2 = v_2 t + \xi_2(r) \quad (2.2)$$

because we have introduced B_{01} and B_{12} only. We let the string worldsheet (τ, σ) span along t and r directions. For convenience we choose worldsheet coordinates $\tau = t$ and $\sigma = r$. As mentioned above, this can also be understood as departing a pair of long-distance separated monopole-antimonopole. While they depart from each other, drag force is generated due to string tension induced via the bulk geometry. The D1 string action is described by the DBI action:

$$S_{DBI} = -\frac{1}{2\pi\alpha'g_s} \int d\tau d\sigma \sqrt{-\det(g+b)_{ab}}. \quad (2.3)$$

The induced metric g and b -field on the D1 string worldsheet is given by $\det(g+b)_{ab} = \det(G+B)_{\mu\nu} \partial_a X^\mu \partial_b X^\nu$, where $a, b = \tau, \sigma$. G, B and g, b respectively denote the spacetime and induced worldsheet metric and B-field. That is,

$$\begin{aligned} g_{\tau\tau} &= -\mathcal{H}^{-1/2}h + \mathcal{H}^{-1/2}|\vec{v}|^2, & b_{\tau\tau} &= 0, \\ g_{\tau\sigma} &= g_{\sigma\tau} = \mathcal{H}^{-1/2}\vec{v} \cdot \vec{\xi}', & b_{\tau\sigma} &= -b_{\sigma\tau} = E\xi'_1 + H\vec{v} \times \vec{\xi}', \\ g_{\sigma\sigma} &= \mathcal{H}^{1/2}h^{-1} + \mathcal{H}^{-1/2}|\vec{\xi}'|^2, & b_{\sigma\sigma} &= 0, \end{aligned} \quad (2.4)$$

where two vectors on the x_1 - x_2 plane, velocity $\vec{v} = (v_1, v_2)$ and projected direction of string tail $\vec{\xi}' = (\xi'_1, \xi'_2)$, are in generic pointing to different directions. \vec{v} and $\vec{\xi}'$ are opposite directions if the monopole-antimonopole pair are departed from each other, while perpendicular if they are parallel transported. In this paper we only consider the former case.

Therefore the Lagrangian density is given by square root of the following determinant

$$\mathcal{L}^2 = -\det(g+b) = 1 + \frac{h}{\mathcal{H}}|\xi'|^2 - \frac{|\vec{v}|^2}{h} - \frac{|\vec{v} \times \vec{\xi}'|^2}{\mathcal{H}} - (E\xi'_1 + H\vec{v} \times \vec{\xi}')^2. \quad (2.5)$$

2.1 Dragging monopole in the electric field

We now consider a monopole moving in a constant speed with constant electric field $E = B_{01}$. To study an influence of E , we may choose the moving direction of monopole to be in the x^1 direction. In this case, the relevant trajectory of the D1 string is more restricted than in (2.2) and assumed to be

$$x_1 = vt + \xi_1(r), \quad x_2 = 0. \quad (2.6)$$

Then

$$-\det(g+b) = 1 - \frac{v^2}{h} + \alpha\xi_1'^2, \quad \alpha = \frac{h}{\mathcal{H}} - E^2. \quad (2.7)$$

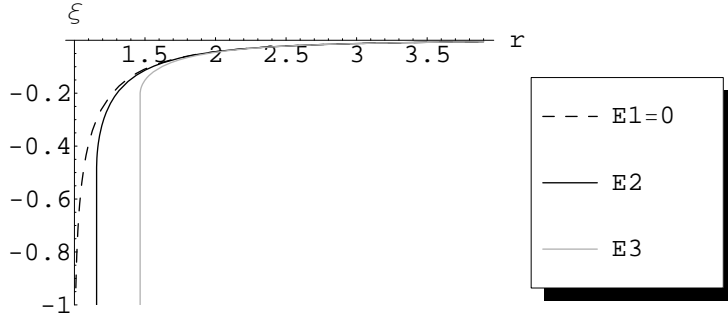


Figure 1: Configuration of string tail v.s. electric B-field ($E1 < E2 < E3$)

With the constant conjugate π_{ξ_1} calculated by $\frac{\partial \mathcal{L}}{\partial \xi_1'}$, one obtains

$$\xi_1' = \pm \pi_{\xi_1} \sqrt{\frac{1 - v^2/h}{\alpha^2 - \alpha \pi_{\xi_1}^2}}. \quad (2.8)$$

The positive sign is chosen for string tail moving behind the quark, i.e. $\xi_1' < 0$. Then the negative π_{ξ_1} can be interpreted as spacetime momentum flow along r -direction on the worldsheet, from the boundary to the black hole. We would like to remark that for the case $\pi_{\xi_1} > 0$, which corresponds to momentum flow from black hole to the boundary, there will be $\xi_1' > 0$ and string is moving ahead the quark. This solution is in fact not physical because classically nothing can get out of black hole. The reality condition for ξ_1' requires the denominator and numerator have a common root at the radius $r_v^4 = \frac{r_H^4}{1-v^2}$, which gives rise to $\pi_{\xi_1}^2 = \alpha(r_v)$. After substituting back, we obtain

$$\xi_1' = v \frac{L^2 r_H^2}{r^4 - r_H^4} \sqrt{\frac{1 - a_e E^2}{1 - b_e E^2}}, \quad a_e = \frac{L^4}{r_H^4} \frac{1}{\gamma^2 v^2}, \quad b_e = \frac{\mathcal{H}}{h}, \quad (2.9)$$

where $\gamma = (1 - v^2)^{-1/2}$ is the inverse Lorentz contraction factor. The reality condition for $\xi_1(r)$ implies a critical value $E_c = a_e^{-1/2}$ for E -field as well as a IR cut-off for r :

$$r > r_{IR} = (r_H^4 + E^2 L^4)^{1/4}, \quad E < E_c. \quad (2.10)$$

We summarize the result in figure 1 which shows the configuration of string tail $\xi_1(r)$ into AdS bulk for different strength of E .

To calculate the flow of energy dissipated into infrared along the string, we first recall the conserved worldsheet current associated with spacetime energy-momentum along x_i is,

$$P_{x_i}^r \equiv \frac{1}{2\pi\alpha' g_s} \frac{\delta \mathcal{L}}{\delta \partial_r x_i} = -\frac{1}{2\pi\alpha' g_s} \pi_{\xi_i}. \quad (2.11)$$

For a particle with momentum \vec{p} moving in a viscous medium and subject to a driving force \vec{f}_1 , we have

$$\dot{\vec{p}}_1 = -\mu \vec{p}_1 + \vec{f}_1, \quad (2.12)$$

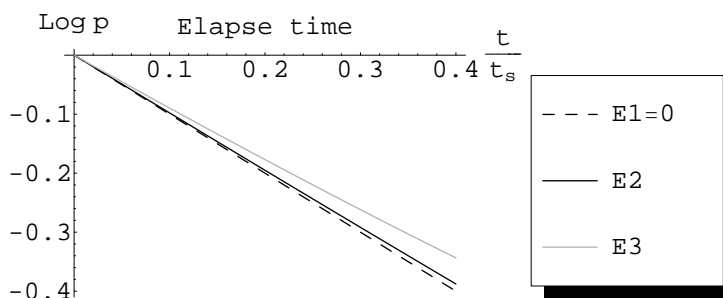


Figure 2: Elapse time v.s. B-field ($E1 < E2 < E3$)

where μ is the damping rate. At the constant speed in our case, we conclude the drag force is given by

$$-f_1 = P_1^r = -\frac{1}{2\pi\alpha'g_s} \sqrt{\gamma^2 v^2 \frac{r_H^4}{L^4} - E^2}. \quad (2.13)$$

We first notice that the viscous force does not appear as a simple linear function of p_1 . The square root shows its nonperturbative nature in the large E limit, however it is also interesting to see the linearized result for small E expansion of (2.13),

$$-f_1 \simeq -\frac{1}{2\pi\alpha'g_s} \frac{r_H^2}{L^2} \gamma v \left(1 - \frac{L^4}{2r_H^4} \gamma^{-2} v^{-2} E^2 + \mathcal{O}(E^4) \right). \quad (2.14)$$

Notice that the correction terms are input as even power of E , thanks to the determinant nature of DBI action. As E is switched off, we reproduce the same result as shown in [6, 7]. It is also instructive to rewrite the drag force in terms of gauge theory parameters, i.e.

$$-f_1 \simeq -\frac{\pi\sqrt{\lambda}}{2g_s} T^2 \gamma v \left(1 - \frac{1}{2\pi^4 \lambda \alpha'^2} T^{-4} \gamma^{-2} v^{-2} E^2 + \mathcal{O}(E^4) \right). \quad (2.15)$$

On the other hand, we may consider a free monopole without driven force. Then equation (2.12) gives us the damping rate or equivalently elapse time constant as inverse μ . In the relativity limit, we have $p_1 = \gamma m v$ for monopole mass m . Here we plot (2.13) in figure 2 to show that the elapse time constant against different strength of E . For vanishing E , one reproduces the *natural* relaxation time constant

$$\tau_0^{-1} = \frac{2g_s m}{\pi\sqrt{\lambda} T^2}, \quad (2.16)$$

however, for nonzero E elapse time constant is *not* a constant. We remark that the appearance of $g_s m$ implies that the effective mass is the same as quark. Therefore we expect this gives us the elapse time constant of the same order as in the F1 string case.

¹Here we use relativistic dispersion relation $p = \gamma m v$ without thermal correction. This correction is expanded by $\Delta m(T)/m$ with a function $\Delta m(T)$ [6]. For heavy particle we can ignore it.

2.2 Dragging monopole in the magnetic field

In this case, we consider the trajectory is given by

$$x_1 = vt + \xi_1(r), \quad x_2 = \xi_2(r), \quad (2.17)$$

with nonzero $H = B_{12}$. Then

$$-\det(g + b) = 1 - \frac{v^2}{h} + \frac{h}{\mathcal{H}}\xi_1'^2 + \beta\xi_2'^2, \quad \beta = \frac{h - v^2}{\mathcal{H}} - H^2v^2. \quad (2.18)$$

The ξ_i' can be obtained by similar calculation as previous section,

$$\xi_1'^2 = \pi_{\xi_1}^2 \frac{\left(1 - \frac{v^2}{h}\right) \beta}{\frac{h}{\mathcal{H}}[(\pi_{\xi_1}^2 - \frac{h}{\mathcal{H}})(\pi_{\xi_2}^2 - \beta) - \pi_{\xi_1}^2 \pi_{\xi_2}^2]},$$

$$\xi_2'^2 = \pi_{\xi_2}^2 \frac{\left(1 - \frac{v^2}{h}\right) \frac{h}{\mathcal{H}}}{\beta[(\pi_{\xi_1}^2 - \frac{h}{\mathcal{H}})(\pi_{\xi_2}^2 - \beta) - \pi_{\xi_1}^2 \pi_{\xi_2}^2]},$$

for two conserved quantities π_{ξ_1} and π_{ξ_2} . Then the reality condition gives

$$\pi_{\xi_1}^2 = \frac{r_H^4}{L^4} \frac{v^2}{1 - v^2}, \quad \pi_{\xi_2}^2 = 0, \quad (2.19)$$

at the end, we have

$$-f_1 = -\frac{1}{2\pi\alpha'g_s} \frac{r_H^2}{L^2} \gamma v_1,$$

$$f_2 = 0. \quad (2.20)$$

Notice that there is no drag force along x_2 -direction and the B_{12} has no effect on the motion along v_1 at all. This may surprise us at first glance. However, it could justify the viscous property of drag force, which is simply against the movement. Therefore that $f_2 = 0$ in fact agrees with a vanishing v_2 .

Here we would like to remark about the case of $\vec{v} \perp E$ which we left in the previous subsection. The analysis is similar to the one in this subsection. We find that there are two solutions. First solution gives $\pi_1 = 0$ therefore we have no drag force along the moving direction. Second case gives $\pi_2 = 0$. In this case we have no effect of E . Therefore the situation given in (2.6) is sufficient to get relevant result.

3. Heavy quark probe in QGP

In this section we will calculate the drag force in hot plasma with large external B-field by using the gravity description for non-commutative supersymmetric Yang-Mills theory [12].² We consider a fundamental string whose endpoints represent a quark and an anti-quark

²Similar calculations on the same background have been done in [13], but at zero temperature.

and use the quark as a probe in hot non-commutative SYM plasma. The dynamics of the fundamental string is governed by Nambu-Goto action³

$$S = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{-\det(\eta^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu G_{\mu\nu})}. \quad (3.1)$$

Here the background metric $G_{\mu\nu}$ is deformed by a external B-field, which is given in the string frame [12];

$$\begin{aligned} ds^2 &= \mathcal{H}^{-1/2} [-h dt^2 + dx_1^2 + k^{-1}(dx_2^2 + dx_3^2)] + \mathcal{H}^{1/2} h^{-1} dr^2 + L^2 d\Omega_5^2, \\ B_{23} &= B_\infty \frac{b^4 r^4}{k}, \quad B_\infty = \frac{\alpha'}{\theta}, \\ e^{2\phi} &= \hat{g}_s^2 k^{-1}, \\ A_{01} &= \frac{\theta}{\alpha' \hat{g}_s} \frac{r^4}{L^4}, \quad F_{0123r} = \frac{1}{\hat{g}_s L^4} \frac{\partial_r(r^4)}{k}, \\ \mathcal{H} &= \frac{L^4}{r^4}, \quad h = 1 - \frac{r_H^4}{r^4}, \quad k = 1 + b^4 r^4, \quad b^2 = \frac{\theta}{\alpha' L^2}, \end{aligned} \quad (3.2)$$

where \hat{g}_s and θ are kept finite under the decoupling zero slope limit. Here \hat{g}_s is the string coupling constant, θ measures the non-commutativity on the D3-brane worldvolume and $L^4/\alpha'^2 = 4\pi\hat{g}_s N$ corresponds to the 't Hooft coupling of the non-commutative SYM theory. The background provides a supergravity dual description of non-commutative Yang-Mills theory at finite temperature.⁴

3.1 Dragging quark in non-commutative space

The magnetic field here points to x^1 -direction, and we have rotational symmetry in the x^2 - x^3 plane, thus we set a frame in which a quark moving along the x^2 -direction which is perpendicular to the magnetic field. Equivalently we consider a quark moving in the non-commutative plane (x^2 - x^3 plane).⁵ The string worldsheet can be written

$$y(t, r) = vt + \zeta(r). \quad (3.3)$$

Plugging this into the string action (3.1), we obtain

$$S = -\frac{1}{2\pi\alpha'} \int dt dr \sqrt{1 - \frac{v^2}{hk} + \frac{h}{\mathcal{H}k} \zeta'^2}. \quad (3.4)$$

One can easily find the string configuration in the r -direction has a derivative:

$$\zeta'(r) = \pm \frac{\mathcal{H}k}{h} \pi_\zeta \sqrt{\frac{hk - v^2}{hk - \mathcal{H}k^2 \pi_\zeta^2}} \quad (3.5)$$

³The endpoints of string can couple to the B-field through the sigma model action. Here we simply neglect the term because the coupling turns out to merely introduce the Lorentz force to the end point quark in the case of constant field.

⁴This non-commutative theory has an interpretation as the SYM with large and constant B field without non-commutativity [14]. However we do not consider such a commutative limit here.

⁵In the case of a quark moving parallel to the magnetic field or perpendicular to the non-commutative plane, the calculation is same as in the case of no B-field.

where π_ζ is the canonical momenta conjugate to ζ and is a constant. The reality condition determines the constant as

$$\pi_\zeta = \frac{v}{L^2} \frac{r_v^2}{1 + b^4 r_v^4} \quad (3.6)$$

where

$$r_v^4 = \frac{1}{2b^4} \left[-(1 - v^2 - b^4 r_H^4) + \sqrt{(1 - v^2 - b^4 r_H^4)^2 + 4b^4 r_H^4} \right]. \quad (3.7)$$

The drag force is given by this constant as

$$-f_2 = P_2^r = -\frac{1}{2\pi\alpha'} \pi_\zeta. \quad (3.8)$$

It is interesting to see the behavior in the limit of small and large br_H , which corresponds to the limit of small and large θ respectively. In the case of small br_H , r_v can be expanded as

$$r_v^4 = \frac{r_H^4}{1 - v^2} \left(1 - \frac{v^2}{(1 - v^2)^2} b^4 r_H^4 \right) + O(b^8 r_H^8). \quad (3.9)$$

Thus we obtain

$$-f_2 = -\frac{1}{2\pi\alpha'} \frac{r_H^2}{L^2} \frac{v}{\sqrt{1 - v^2}} \left[1 - \frac{2 - v^2}{2(1 - v^2)^2} b^4 r_H^4 + O(b^8 r_H^8) \right]. \quad (3.10)$$

In a non-relativistic approximation which is valid for small velocity, one can assume $p = mv$ and the drag force becomes

$$-f_2 \simeq -\frac{\pi}{2} \sqrt{\lambda} T^2 \frac{p_2}{m} \left[1 - \pi^4 T^4 \lambda \theta^2 + O(\theta^4) \right], \quad (3.11)$$

where we have used the relations $b^4 r_H^4 = \pi^4 T^4 \lambda \theta^2$. The relaxation time derived from this becomes

$$\tau_0 = \frac{2m}{\pi \sqrt{\lambda} T^2} (1 + \pi^4 T^4 \lambda \theta^2 + O(\theta^4)), \quad (3.12)$$

which shows the non-commutative nature makes the medium less viscous.

Even in relativistic velocity, equation (3.10) is valid if $b^4 r_H^4 \ll (1 - v^2)^2$. We use relativistic dispersion relation $p = \gamma mv$, then the force behaves as

$$-f_2 \simeq -\frac{\pi}{2} \sqrt{\lambda} T^2 \frac{p_2}{m} \left[1 - \frac{\pi^4 T^4 \lambda \theta^2}{2m^4} p_2^4 + O(\theta^4) \right]. \quad (3.13)$$

Dissipation of energy becomes milder than in lower momentum region. The above analysis shows that as θ is turn off the result reduces to the one obtained in [6, 7].

On the other hand, for large br_H or θ we have

$$b^4 r_v^4 = b^4 r_H^4 \left(1 + \frac{v^2}{b^4 r_H^4} + O(b^{-8} r_H^{-8}) \right) \quad (3.14)$$

therefore

$$\begin{aligned}
 -f_2 &= -\frac{1}{2\pi\alpha'} \frac{v}{L^2} \frac{1}{b^4 r_H^2} \left(1 - \frac{2+v^2}{2b^4 r_H^4} + O(b^{-8} r_H^{-8}) \right) \\
 &\simeq -\frac{1}{2\pi^3 T^2 \sqrt{\lambda} \theta^2} \frac{p_2}{m} \left[1 - \frac{1}{\pi^4 T^4 \lambda \theta^2} + O(\theta^{-4}) \right]
 \end{aligned}
 \tag{3.15}$$

for small velocity. For relativistic velocity and $b^4 r_H^4 \gg (1-v^2)^2$, the first line in large $b^4 r_H^4$ expansion (3.15) is valid. Using the relativistic dispersion relation then we have

$$-f_2 \simeq -\frac{1}{2\pi^3 T^2 \sqrt{\lambda} \theta^2} \left[1 - \frac{3}{2\pi^4 T^4 \lambda \theta^2} + O(\theta^{-4}) \right].
 \tag{3.16}$$

In this limit we find constant friction. It is interesting to see the dependence of the 't Hooft coupling and temperature is inverted relative to the small non-commutative case.

A comment concerning to the relation to the ordinary Yang-Mills theory with magnetic field is in order. The non-commutative theory on D-brane allows another interpretation as ordinary theory with (large) constant electromagnetic field on the D-brane. According to this interpretation, our results imply that the constant magnetic field affects thermal plasma to decrease the drag force for both large and small velocity. In other words, a large magnetic field may decrease the effective viscosity of QGP.

4. Summary and discussion

We have investigate the issue on the drag force in thermal SYM with B-field or NCSYM via AdS/CFT with appropriate backgrounds. We have used a quark and a monopole as probes to measure the drag force in the thermal medium.

For the monopole probe we have studied the problem with AdS₅ Schwartzchild black hole background with constant B-field and use the DBI action as effective action of D1 brane whose endpoint is the monopole to calculate the drag force. We considered separately the two cases in which either a constant B_{01} or B_{12} is turned on. In the case of turning on B_{01} , we evaluate the drag force in the x^1 direction by taking the monopole moving along this direction, whereas in the B_{12} case the motion of the monopole is perpendicular to the magnetic force, and we allow the drag force span in both x^1 and x^2 directions. We found for electric case the effects of the B-field reduce the drag force regardless of its sign. This suggests turning on electric B-field effectively weakens the viscosity of the medium to a monopole. To our surprise, the magnetic B-field do not have any effect on the drag force of a monopole, which somehow reveals its nature of viscous force.

For the quark probe we use Nambu-Goto action describing a fundamental string, whose endpoint is the quark, with the background containing the effect of constant B-field which results in the non-commutative super Yang-Mills theory or ordinary SYM theory with constant field strength on the boundary. We have studied θ and θ^{-1} expansions of the drag force of quark probe with small and large velocity. Again, the result implies that the effect of the constant B-field or non-commutativity makes the medium less viscous.

Thus we suggest that static magnetic field may decrease the effective viscosity of QGP to a quark. The natural guess is that even in the situation of real QCD a constant magnetic field makes thermal plasma less viscous.

We leave several problems as future works. One of them is to investigate effects of static electric field by introducing B_{01} in the Maldacena-Russo black hole background. This background is already known as the supergravity dual of non-commutative open string (NCOS) theory and the metric is given by [15]

$$\frac{ds^2}{\alpha'} = f^{-1/2}G(-hdt^2 + dx_1^2) + f^{-1/2}(dx_2^2 + dx_3^2) + \frac{1}{\alpha'^2}f^{1/2}(h^{-1}dr^2 + r^2d\Omega^2),$$

$$f = 1 + \frac{\alpha'^2 L^4}{r^4}, \quad G = \frac{r^4}{\alpha'^2 L^4} f^{-1}, \quad h = 1 - \frac{r_H^4}{r^4}. \quad (4.1)$$

where the factor G represents the effect of B_{01} . We can easily calculate the drag force with ansatz: $x^1 = vt + \xi(r)$ and obtain

$$-f_1 = -\frac{1}{2\pi} \sqrt{1 + \frac{1}{1-v^2}(\pi T_H)^4 \lambda} \frac{v}{\sqrt{1-v^2}} (\pi T_H)^2 \sqrt{\lambda}. \quad (4.2)$$

The effect of electric field introduce a square root factor $\sqrt{1 + \frac{1}{1-v^2}(\pi T_H)^4 \lambda}$ thus increases drag force. This is contrast to the case of magnetic field where drag force decreases. The quark moving in hot NCOS which is not a field theory. It is interesting to identify the medium to be described by NCOS.

At last, It remains to study the monopole case by considering DBI action with Maldacena-Russo background. To estimate jet-quenching parameters in the setup investigated in the present paper may also be an interesting problem as well.

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